Dynamic Analysis of an In-Flight Refueling System

J. Eichler*
Ben Gurion University of the Negev, Israel

As part of the design of an in-flight refueling system at Israel Aircraft Industries Ltd. (IAI), a dynamic analysis was performed. The IAI system has twin refueling pods set far out on the wings. Consequently, the effects of wing vibration and vortex become important, together with the usual disturbance of vertical wind gusts. The paper presents the derivation and solution of the nonlinear partial differential equations. The solution was obtained in closed form for sine-wave gust disturbances and numerically for both sine wave and pulse-type vertical gusts and wing vibration. The vortex problem was studied separately using a simplified model. Two designs and several flight conditions were considered. The paper describes the entire study and conclusions. Flight tests of the system are now being conducted and early results corroborate the vortex conclusions obtained in the analysis.

Nomenclature

C_l = hose lift constant
C_2 = hose drag constant
C_DS = drogue drag constant times characteristic area, ft^2
$\mathfrak{C}$ = $(=K_l+3\rho_H g/2)$ in static trail equation
d = diameter of hose, in.
$\bar{c}$ = mean aerodynamic chord, in.
FF = $-W/gQ$ intermediate value
g = acceleration of gravity, ft/s^2
k_l = $\rho C_l(d/12)V^2$ in hose lift equation
p = lift per unit length of hose, lb/ft
P = drogue lift, lb
q = drag per unit length of the hose, lb/ft
Q = drogue drag, lb
r = input wing vibration frequency
s = distance along the hose, ft
s_0 = length of the hose, ft
T = tension
t = time, s
V = velocity of the aircraft, knots
w = input wind disturbance, fps
w_{de} = magnitude of input wind function, fps
w_0 = w_{de}/V velocity normalized wind gust magnitude
W = weight of drogue, lb
x = axis along horizontal plate at level of the pod, ft
y = vertical axis from pod down, positive down, ft
Y_{end} = distance of drogue below x axis, ft
α = angle of attack
ρ = air density, slugs/ft^2
ρ_H = hose weight per unit length, slugs/ft
$\Delta_{y_{\text{end}}}$ = $y_{\text{end}}(\text{new})-y_{\text{end}}(\text{nominal})$
Δ_{angle} = angle at drogue (new)- angle at drogue (nominal)
δ_x = small perturbation of x
δ_y = small perturbation of y

I. Introduction

AS part of the development of an in-flight refueling system at Israel Aircraft Industries Ltd. (IAI), a design study was conducted of the dynamics of the system. The IAI design sets twin refueling pods far out on the wing. This brings into play the disturbances from the wing vibration and trailing vortices. These were not important in prior designs set close to or on the body.

The design goal of an in-flight refueling system is optimum system performance. This requires that the system minimize drogue motion for given flight conditions of speed and altitude and in the presence of disturbances caused by wind gusts, wind vibration, and vortex. Thus, the problem is defined by giving a set of desired flight conditions, a worst-case model of wind gust disturbances, wing vibration information, and vortex data. The designer must then choose design variables, such as drogue weight and drag, hose length, and width and weight per unit length, that will obtain minimum drogue motion or at least keep it within some tolerance. Tolerance may be specified by the ± diameter of the drogue traverse in a second. The mathematical model of this system is a set of nonlinear partial differential equations.

The problem is treated by first solving the nonlinear unforced mathematical model to obtain the static trail position of the hose and subsequently developing and solving perturbation equations about the static solution. The perturbation equations are linear hyperbolic partial differential equations with variable coefficients. These were first solved analytically for certain conditions and then numerically using finite differences. The sensitivity of the solution to some design variables and problem variables were also calculated.

The correctness of the solution is demonstrated by comparing the analytic and numerical method solutions. The correctness of the model itself is demonstrated by showing that the same order of magnitude effect is obtained when using a very simplified model. Preliminary flight tests of the system show the vortex analysis to be correct and indicate that the general conclusions regarding wind gusts and wing vibration hold. It is anticipated that fully instrumented flights will yield more test comparison to the theoretical solution.

This paper presents the mathematical model, its solution, and the use of the model for evaluation of two alternate designs and sensitivity calculations that will permit optimization of dynamic performance. Analysis of the vortex problem is also given.

II. Definition of the Problem

The dynamic problem of in-flight refueling contains a number of phases as follows: the tanker attaining flight conditions for fueling; letting out the hose/drogue system; the transient motion of the hose/drogue system until it is fully out and stabilized; subsequent disturbances due to vertical wind gusts, wing vibration, and vortex; hook-up to the drogue, etc. This study is concerned with that phase starting after the transient of set-up of the system has passed and continuing until the phase of hook-up; i.e., we consider the effect on a set-up hose/drogue system of the disturbance due to vertical wind gusts, wing vibration, and vortex effects.

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*Associate Professor, Mechanical Engineering Department.

The tanker plane is assumed to be in level flight at a constant velocity. The main design question is whether the vertical motion of the drogue will allow hook-up to be done easily. Ideally, if the drogue's vertical motion does not exceed ± its own diameter, the plane being fueled will be able to insert its probe easily into a section of the cone of the drogue and attain hook-up.

The possibility of attaining a desirable dynamic response to disturbances will vary for different flight conditions of altitude and speed, and will of course depend on the design parameters of the hose drogue system. We assume that the wing vibration spectrum is fixed and therefore not a design variable and that a worst-case vertical gust is defined for the design. Furthermore, we assume that the vortex effects of the plane are given. The flight conditions are also prescribed. The changeable design parameters of the system and their nominal values are shown in Table 1.

In addition to selecting one of the two hoses, the variables hose length, drogue weight, and drogue drag were studied to find their effect on changes about the nominal. The possibility of changing weight/length of the hose was also considered at first but later rejected because of the anticipated difficulty in making this design change.

The entire set of flight conditions are shown in Table 2. The static hose position was studied for all flight conditions. The effect of wind gusts was studied for cases 1 and 11, i.e., sea level, 200 knots and 25,000 ft, 346 knots. Case 11 was also the flight condition for the study of wing vibration effects.

The specific disturbances considered were: the actual vortex of the plane; vertical wind gusts, where the input wind magnitude was 61.3 fps for 25,000 ft altitude and 66 fps for sea level altitude; and wing vibrations from 1 Hz to 3 Hz. The forms of the vertical gusts were

a) $$w=\frac{w_{de}}{2}\left(\sin\frac{2\pi}{25\bar{c}}Vt\right)$$

and

b) $$w=\frac{w_{de}}{2}\left(1-\cos\frac{2\pi}{25\bar{c}}Vt\right)\quad \text{a single pulse}$$

Table 1 Design parameters of the system and their values

	Hose 1	Hose 2
Outer diameter, in.	2.0	2.94
Weight/length (full), lb/ft	1.858	4.023
Length of hose, ft	50.	75.
Drogue weight, lb	52.8	
Drogue drag (C_DA), ft^2	2.2	

Table 2 Static trail position results

	Flight conditions			Angle at drogue,
Case	Altitude, ft	Speed, knots	y_{end}, ft.	deg.
1	S.L.	200[a]	11.3	11.3
2	S.L.	265	7.4	6.6
3	S.L.	330	5.2	4.4
4	10,000	200	13.8	14.9[b]
5	10,000	250[a]	10.2	9.9
6	10,000	300	7.7	7.0
7	10,000	330	6.6	5.8
8	25,000	205	18.1	22.0[b]
9	25,000	250	14.3	15.6[b]
10	25,000	292	11.6	11.7
11	25,000	346[a]	9.1	8.6
12	32,000	301	12.8	13.5

[a] Used for sensitivity studies.
[b] Exceeds assumed limit for constant hose C_D (12 deg).

The value of w_{de} was determined to be 66 fps for altitude 0-20,000 ft, and to vary linearly from 66 fps to 38 fps for altitudes between 20,000 to 50,000 ft; $\bar{c}$, the mean aerodynamic chord, was taken as 283 in. The basic input is then 50 halfchords wavelength. The wing vibration input was taken as sinusoid and of unit magnitude for a range 1-3 Hz. Of particular interest were the frequencies 1.3 Hz and 2.1 Hz, where peak input was anticipated based on structural analysis.

In summary, the problem was to study the hose/drogue systems' dynamic response for various given flight conditions for prescribed disturbances and for two different designs with further variations of design parameter possible. The purpose was to determine a set of design variables that would give "best" dynamic stability. Furthermore, it was desired to know the sensitivity of the dynamic response to variation of system and design parameters to enable future optimization of response.

III. Mathematical Model

Within the framework of the problem definition, the equations of motion of the hose/drogue system were derived. Consider the geometry of the problem, in a vertical plane (Fig. 1). Summing the forces about the y and x axes yields

$$\rho_H\frac{\partial^2 y}{\partial t^2}\,ds+p\,ds+T\frac{\partial y}{\partial s}-\rho_H g\,ds - \left(T+\frac{\partial T}{\partial s}ds\right)\left(\frac{\partial y}{\partial s}+\frac{\partial^2 y}{\partial s^2}ds\right)=0 \tag{1}$$

$$\rho_H\frac{\partial^2 x}{\partial t^2}\,ds-q\,ds+T\frac{\partial x}{\partial s} - \left(T+\frac{\partial T}{\partial s}ds\right)\left(\frac{\partial x}{\partial s}+\frac{\partial^2 x}{\partial s^2}ds\right)=0 \tag{2}$$

The boundary condition of these equations at the pod end is zero if we consider wind gusts alone, and $\sin\omega t$ if we consider wing vibration alone. At the drogue end, we require that the resultant forces on the end of the hose be in the same direction as the resultant forces acting on the drogue.

The model used for the drag/length q and the lift/length p of the hose are

$$q=\rho C_2\,(d/12)\,V^2$$

$$p=\rho C_1\,(d/12)\,V^2\alpha^2$$

where $C_1=0.57$ and $C_2=0.006$, based on Refs. 1 and 2.

Assumptions used in solving the equations of the model include the following:

1) In the static trail position equations the hose drag coefficient was assumed to be constant. This is true up to about 12 deg angle of attack.

2) In developing the equations for wind gust and wing vibration effects, a perturbation analysis was done. This, of course, assumes small changes about the static trail position.

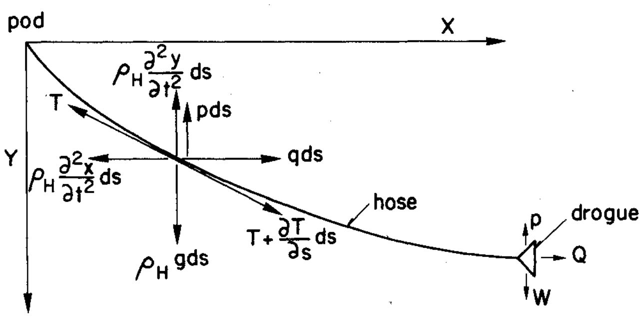

Fig. 1 Geometry.

3) The tension in the hose was approximated to be constant and equal to its value at ½ the hose length for the analytical solution. This was later seen to be a reasonable assumption.

4) Effects of transients were disregarded for the sine wave part of the wind gust study and only the steady-state response was reported.

IV. Method of Solution

The approach to solving Eqs. (1) and (2) for the various flight conditions, disturbances, design values, etc., was to first solve the equation for the static trail position and then to derive a set of perturbation equations for disturbances about the nominal static trail position. The solution of the static trail equation is given first. Considering Eqs. (1) and (2) and ignoring the ds^2 terms, we obtain the static trail equations:

$$p\mathrm{d}s-\rho_H g\mathrm{d}s-T\frac{\partial^2 y}{\partial s^2}\mathrm{d}s-\frac{\partial T}{\partial s}\frac{\partial y}{\partial s}\mathrm{d}s=0$$

$$q\mathrm{d}s-\frac{\partial T}{\partial s}\frac{\partial x}{\partial s}\mathrm{d}s=0$$

which gives

$$p-\rho_H g-\frac{\mathrm{d}}{\mathrm{d}s}\left(T\frac{\mathrm{d}y}{\mathrm{d}s}\right)=0 \tag{3}$$

$$q+\frac{\mathrm{d}}{\mathrm{d}s}\left(T\frac{\mathrm{d}x}{\mathrm{d}s}\right)=0 \tag{4}$$

We also have the trigonometric identity

$$\left(\frac{\mathrm{d}x}{\mathrm{d}s}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}s}\right)^2=1 \tag{5}$$

Using the model of the drag and lift for the hose

$$p=K_1(\mathrm{d}y/\mathrm{d}s)^2 \qquad q=\text{constant}$$

gives

$$T(\mathrm{d}x/\mathrm{d}s)=-qs+\text{constant}$$

The boundary condition at the drogue end yields

$$T(\mathrm{d}x/\mathrm{d}s)=Q$$

and since $s=s_0$ at the end, we get

$$Q=-qs_0+C_1$$

This gives

$$C_1=Q+qs_0$$

therefore

$$T(\mathrm{d}x/\mathrm{d}s)=-qs+Q+qs_0$$

or

$$T=\frac{q(s_0-s)+Q}{\mathrm{d}x/\mathrm{d}s}$$

Defining $z=\mathrm{d}y/\mathrm{d}s$ and using Eq. (5), we get from Eq. (3)

$$K_1z^2-\rho_H g-\frac{\mathrm{d}}{\mathrm{d}s}\left\{\frac{q(s_0-s)+Q}{\sqrt{1-z^2}}z\right\}=0$$

defining

$$\text{¢}=K_1+3\rho_H g/2$$

and integrating we get

$$-(1/q)\log\,(qs_0-qs+Q)$$

$$=\frac{1}{\sqrt{+4\rho_H g\,\text{¢}+q^2}}\log\frac{2\,\text{¢}\,z+q-\sqrt{+\eta}}{2\,\text{¢}\,z+q+\sqrt{+\eta}}+\text{constant}$$

where

$$\eta=+(4\rho_H gc+q^2)/q^2$$

and $\sqrt{1-z^2}$ has been taken as $1-½z^2$. The constant is obtained by requiring that the hose line up with the drogue at the end.

$$z=\frac{W-P}{\sqrt{Q^2+(W-P)^2}}\simeq\frac{W-P}{Q}=\frac{W}{Q}$$

since in our case P is negligible small. This gives finally

$$C=\text{constant}=\left(\frac{2\,\text{¢}\,W/Q+q-\sqrt{\eta}}{2\,\text{¢}\,W/Q+q+\sqrt{\eta}}\right)Q^{\sqrt{\eta}}$$

and

$$z=\frac{\mathrm{d}y}{\mathrm{d}s}=\frac{(1-\sqrt{\eta})-(1+\sqrt{\eta})}{2C(f-1)}$$

where

$$f=C/(qs_0-qs+Q)^{\sqrt{\eta}}$$

The equation for $\mathrm{d}y/\mathrm{d}s$ was solved numerically using an Euler integration scheme with a step size in s of 0.2 ft. Equation (5) yielded the $\mathrm{d}x/\mathrm{d}s$ equation that was solved numerically in the same manner.

The perturbation equations are derived as follows: Starting with the equations obtained by summing the forces indicated in Fig. 1, we obtain

$$\rho_H\frac{\partial^2 y}{\partial t^2}+p-\rho_H g-\frac{\partial T}{\partial s}\frac{\partial y}{\partial s}-T\frac{\partial^2 y}{\partial s^2}=0$$

We now assume small perturbations about the static trail position, that is,

$$y(s,t)=\bar{y}(s)+\delta_y(s,t)$$

where $\bar{y}(s)$ is the static trail position value. Before deriving the perturbation equations, we consider the two parameters – p, the lift/length, and T, the tension.

Lift/Length p

The induced angle of attack due to a wind gust can be shown to be approximately equal to[1]

$$\sin\alpha=\frac{\partial y}{\partial s}+\frac{\partial y/\partial t}{V}+\frac{w}{V}+\text{higher order terms}$$

Assuming w^2/V^2 is negligible compared to w/V, we retain only the first three terms. If we then linearize p about the static trail position values, we obtain

$$p=(p)_{\text{equil}}+(\mathrm{d}p/\mathrm{d}\alpha)_{\text{equil}}(\alpha-\partial\bar{y}/\partial s)$$

Using our model, $p=K_1\sin^2\alpha$, we obtain

$$\mathrm{d}p/\mathrm{d}\alpha=2K_1\sin\alpha\cos\alpha\simeq 2K_1\sin\alpha$$

and

$$p=K_l\left(\frac{\partial\bar{y}}{\partial s}\right)^2+2K_l\left(\frac{\partial\bar{y}}{\partial s}\right)\left(\frac{\partial y}{\partial s}+\frac{w}{V}+\frac{\partial y/\partial t}{V}-\frac{\partial\bar{y}}{\partial s}\right)$$

inserting

$$\frac{\partial y}{\partial s}=\frac{\partial\bar{y}}{\partial s}+\frac{\partial\delta y}{\partial s}$$

yields, finally

$$p=K_l\left(\frac{\partial\bar{y}}{\partial s}\right)^2+2K_l\left(\frac{\partial\bar{y}}{\partial s}\right)\left\{\frac{\partial\delta y}{\partial s}+\frac{w}{V}+\frac{\partial\delta y/\partial t}{V}\right\}$$

Tension T

Our first assumption is to approximate the tension to be that of the static trail position

$$T(s,t)=\bar{T}(s)$$

Using the development given in the derivation of the static trail position equations yields

$$\bar{T}=\frac{q(s_0-s)+Q}{\mathrm{d}\bar{x}/\mathrm{d}s}$$

Perturbation Equations

With the terms for p and T in hand we return to the original partial differential equation and insert these terms and drop the equilibrium terms to obtain

$$\rho_H\frac{\partial^2\delta y}{\partial t^2}+\frac{2K_l}{V}\left(\frac{\partial\bar{y}}{\partial s}\right)\left(\frac{\partial\delta y}{\partial t}\right)$$
$$-\left\{\frac{\partial\bar{T}}{\partial s}-2K_l\left(\frac{\partial\bar{y}}{\partial s}\right)\right\}\frac{\partial\delta y}{\partial s}-\bar{T}\frac{\partial^2\delta y}{\partial s^2}=-2K_l\left(\frac{\partial\bar{y}}{\partial s}\right)\frac{w}{V}$$

Boundary Conditions

The boundary conditions are determined by the following: 1) the hose at the pod must follow the wing motion; and 2) the hose at the drogue must have the angle indicated by the resultant forces on the drogue. These give

$$\text{at } s=0 \qquad y=y'(t) \qquad \text{or} \qquad \delta y=y'(t)$$

and

$$\text{at } s=s_0 \qquad \frac{\partial\delta y}{\partial s}=-\frac{W}{gQ}\frac{\partial^2\delta y}{\partial t^2}$$

The perturbation equations were solved for vertical wind gusts in two ways, analytically (i.e., closed form) and numerically. For the closed-form solution, a sine wave disturbance was used, and for the numerical method a sine wave was used and also a single pulse of the shape $(1-\cos)$. To allow a closed-form solution, $\bar{T}$, $\mathrm{d}\bar{T}/\mathrm{d}s$, and $\mathrm{d}y/\mathrm{d}s$ were approximated as constant and equal to their value at $s_0/2$, i.e., halfway down the hose.

The closed-form solution for a sine input is of the form

$$\delta_y(s,t)=g_l(s)\sin rt+g_2(s)\cos rt$$

where $g_l(s)$ and $g_2(s)$ are function of the boundary conditions and eigenvalues of the system. This method of solution is very complicated and entailed exhausting algebraic manipulation. It is only applicable to the sine input and only gives the steady-state solution. Consequently, a numerical solution was also obtained.

If we write the perturbation equation as

$$A\frac{\partial^2\delta y}{\partial t^2}+B\frac{\partial\delta y}{\partial t}+C\frac{\partial\delta y}{\partial s}+D\frac{\partial^2\delta y}{\partial s^2}=E\frac{w}{V}$$

when

$$A=\rho_H \qquad B=\frac{2K_l}{V}\frac{\mathrm{d}y}{\mathrm{d}s} \qquad K_l=\rho\frac{d}{12}V^2C_l$$

$$C=-\left(\frac{\partial\bar{T}}{\partial s}-2K_l\frac{\mathrm{d}y}{\mathrm{d}s}\right) \qquad D=-\bar{T} \qquad E=-2K_l\frac{\mathrm{d}y}{\mathrm{d}s}$$

then

$$\delta_{i,j+l}=AA\,\delta_{i,j}+BB\delta_{i,j-l}+CC\delta_{i+l,j}+DD\delta_{i-l,j}$$

where i is the space step and j is the time step, and where AA, BB, CC, DD are

$$AA=\left(\frac{2A}{k^2}+\frac{2D}{h^2}+\frac{B}{k}+\frac{C}{h}\right)\Big/\left(\frac{A}{k^2}+\frac{B}{k}\right)$$

$$BB=-\frac{A}{k^2}\Big/\left(\frac{A}{k^2}+\frac{B}{k}\right)$$

$$CC=-\left(\frac{D}{h^2}+\frac{C}{h}\right)\Big/\left(\frac{A}{k^2}+\frac{B}{k}\right)$$

$$DD=-\frac{D}{h^2}\Big/\left(\frac{A}{k^2}+\frac{B}{k}\right)$$

and the step size along the hose is h, and in time k. The boundary conditions translate into

$$\delta(0,j)=\sin(\omega\cdot j\cdot k)$$

for wing vibration and for wind gusts

$$\delta(0,j)=0 \qquad \text{for all } j$$

also in all cases $\delta(n,j)$, that is at the drogue end of the rope is given by

$$\delta_{n,j+l}=\frac{-k^2}{h\cdot FF}(\delta_{n-l,j}-\delta_{n,j})+2\delta_{n,j}-\delta_{n,j-l}$$

where for wind gusts we have the forcing term added to yield

$$\delta_{i,j+l}=(\delta_{h,j+l})_{\text{above}}+\frac{E}{[(A/k^2)+(B/k)]}$$
$$\times\left(\frac{w_0}{2}\right)\sin\frac{2\pi V}{4G\bar{c}/2}\left(j\cdot k-\frac{ih}{V}\right)$$

The equations were solved on a CDC 6600 computer, and care was taken in selecting the ratio of h and k to insure stability. Validation of the model and verification of the solution was done by comparison with rough calculations based on a lumped model and by comparing closed-form and numerical solutions (see Table 7).

V. Results

The study was conducted in several stages. The first stage was the development of the math model for the static trail position of the hose. This model was solved for the 12 nominal operating points listed in Table 2. A sensitivity study was also conducted, using this model, to find the variation in static trail position casued by variations of the following design parameters: drogue aerodynamic constant, drogue weight, hose length, and type of hose (weight per unit length, outside diameter). The sensitivity was studied at three operating points: sea level, 200 knots; 10,000 ft. altitude, 250 knots; and 25,000 ft altitude, 346 knots.

The second phase of the study was to calculate the anticipated effect of vortex from the wingtip on the hose drogue system. This was done with a crude math model. Use of refined models was not required, as the effects proved to be negligible.

The third phase was the development of the perturbation partial differential equations. Wing vibration effects were studied by calculating the ratio of drogue motion to wing vibration amplitude for frequencies from 1 Hz up to 3 Hz. A sensitivity study was also conducted to determine which design parameters effect the amplitude/frequency curves. The effect of vertical wind gusts was also studied for two operating points: sea level, 265 knots, and 25,000 ft altitude, 346 knots. A sensitivity study was also conducted to check the effect of hose length, drogue weight, and drogue aerodynamics constant. In addition, two different hoses were studied. Table 2 summarizes the static trail position end values for the 12 flight conditions and hose 1.

The drop in hose at the drogue below the horizontal plane (y end) ranges between 5.2 ft for case 3 to 18.1 ft for case 8. As expected, the lower air density at higher altitudes yields less lift and consequently larger y_{end} than at lower altitudes. The variation of cases 1, 4, and 8 from 11.3 ft to 13.8 ft to 18.1 ft shows this expected trend. Similarity, higher speed at the same altitude gives lower y_{end} values due to higher lift forces. At 25,000 ft the range is from 18.1 ft at 205 knots to 9.1 ft at 346 knots. The angle at the drogue varies accordingly with the y_{end}. Figure 2 gives graphs of the static trail position for two cases.

Sensitivity of the static trail position to various design parameters was studied. The term sensitivity is defined

$$\text{sensitivity} = \frac{[y_{end}(\text{new}) - y_{end}(\text{nom})]/y_{end}(\text{nom})}{[\text{parameter (new)} - \text{parameter (nom)}]/\text{parameter (nom)}}$$

where nom indicates nominal. The results of the sensitivity

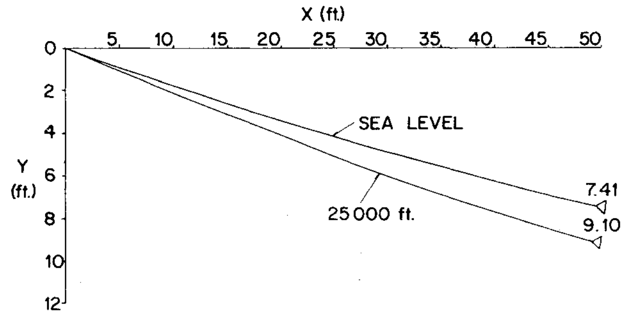

Fig. 2 Static hose position at sea level, 265 knots and 25,000 ft, 346 knots.

study are for use in case of design changes. The data will indicate the design performance to be anticipated for a given design parameter change.

The sensitivity results are summarized in Tables 3a and 3b. Table 3a covers hose 1 and 3b hose 2. Table 3 indicates that hose 2 produces a lower y_{end} than hose 1 by about 2 ft throughout. The sensitivity to hose length is largest and to drogue weight least, whereas C_DS (drogue drag) is also a rather insensitive parameter. The angle at y_{end} remains constant while varying the hose length, for both hoses, demonstrating that the hose is essentially straight at the end. The sensitivity of the longer hose design, hose 2, was only calculated for length variation. It is assumed that hose 1 sensitivities hold approximately for the larger hose as well.

Dynamic Effects

Three effects were studied: wing vortex, wing vibration, and wind gusts. The problem was to determine whether any instability would occur under prescribed conditions and to ascertain the motion of the drogue for prescribed flight conditions and disturbances. Although all three effects occur together, they were analyzed separately. The conclusion section deals with estimating the overall effect of all the disturbances acting at once.

A "worst-case" analysis was made of the possible effect of wingtip vortex on the hose and drogue during in-flight refueling. The analysis discloses that the vortex effect alone

Table 3a Static trail position sensitivity (hose 1)

	Flight conditions		Varied					
Case	Altitude, ft	Speed, knots	Parameter	From	To	Δy_{end}	Δ_{angle}	y_{end} sensitivity
1	S.L.	200	C_DS	2.2	1.9	0.70	1.4	−0.45
2					2.5	−0.70	−1.2	−0.45
3					2.7	−1.10	−1.9	−0.42
4	10,000	250			1.9	0.7	1.3	−0.50
5					2.5	−0.7	−1.1	−0.50
6					2.7	−1.1	−1.7	−0.47
7	25,000	346			1.9	0.7	1.1	−0.56
8					2.5	−0.6	−1.0	−0.58
9					2.7	−1.0	−1.5	−0.48
10	S.L.	200	Drogue weight	52.8	60.	0.5	1.2	+0.35
11					65.	0.9	2.0	0.34
12					70.	1.2	2.8	0.32
13	10,000	250			60.	0.5	1.0	0.36
14					65.	0.8	1.7	0.34
15					70	1.1	2.5	0.33
16	25,000	346			60.	0.4	0.9	0.32
17					65.	0.8	1.5	0.38
18					70.	1.1	2.1	0.37
19	S.L.	200	Hose length	50.	40.	−2.5	0	1.11
20					45.	−1.2	0	1.06
21	10,000	250			40.	−2.3	0	1.13
22					45.	−1.1	0	1.08
23	25,000	346			40.	−2.0	0	1.10
24					45.	−1.0	0	1.10

will not produce instability of the hose drogue, and in fact there is a wide margin of engineering safety in the design. The analysis was done using a gross lumped parameter model[3] and 300,000 lb weight aircraft at various altitudes from sea level to 25,000 ft, and speeds from 200 to 346 knots.

The side force model of the drogue was based on an estimate of effective area 1.12 ft^2. The drag coefficient was taken between 1.2 and 1.56, thus yielding a $C_D S$ value of 1.34 to 1.75 ft^2. All the calculations of the location of the maximum velocity of the vortex place it at a radial distance of about 0.8 ft. This was conservatively taken as if measured from the pod. Furthermore, no decay was assumed for the velocity with downstream distance as the distance is only about 50 ft. A radial decay was taken as $1/r$, that is,

$$\frac{v}{v_0} = \frac{a}{r}$$

where v is vortex velocity at a distance r, a is radial distance to the maximum vortex velocity, and v_0 is the maximum vortex velocity. In the cases studied, r was taken as equal to y_{end}. Table 4 summarizes the calculations. A final comment: the velocity needed to sustain the 52.8 lb drogue at y_{end} is 159 fps at sea level, 185 fps at 10,000 ft altitude and 237.6 fps at 25,000 ft altitude. These figures do not consider the hose weight which is 116.5 lb. This leads to the conclusion that the aerodynamic force on the drogue due to the trailing vortices is negligible, even when making a conservative estimate of the drag characteristics of the drogue.

The perturbation equations give the changes δ_y and δ_x from the static trail position due to input disturbances. In the case of wing vibration, the equations were solved for frequencies for 1 Hz to 3 Hz. The results are given in terms of output amplitude per unit input amplitude, i.e., gain. Of particular interest are frequence of 1.3 Hz and 2.1 Hz where peak input is anticipated. The flight conditions were 25,000 ft altitude and 346 knots speed. Plotted in Fig. 3 are the results for case I, hose 1 nominal conditions; case II, shortened hose length from 50 to 45 ft; plus a few points from case III showing the effect of increasing drogue weight from 52.8 to 57.8 lb. In-

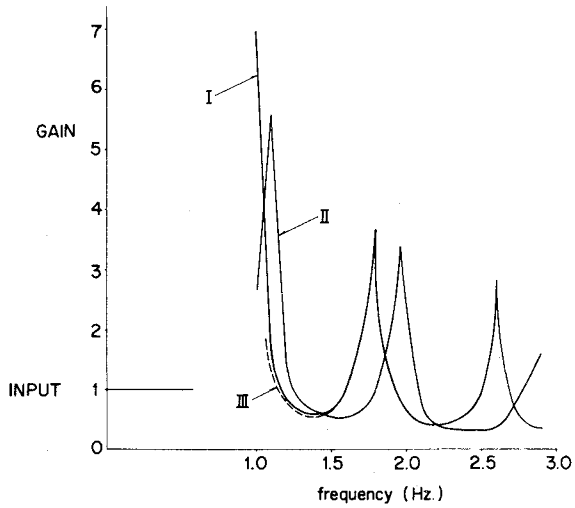

Fig. 3 Wing vibration response, gain vs frequency for unit input.

Table 3b Static trail sensitivity (hose 2)

Case	Flight conditions: Altitude, ft.	Speed, knots	Parameter varied	s_0, ft	y end, ft.	Angle, deg.	Sensitivity
1	S.L.	200	New hose (hose 2)	50	13.7	12.7	1.06
2	10,000	250		50	12.5	11.2	1.09
3	25,000	346		50	11.3	9.7	1.11
4	S.L.	200	New hose lengths (hose 2)	60	16.6	12.7	1.08
5				65	18.2	12.7	1.06
6				70	19.7	12.7	1.06
7				75	21.2	12.7	[a]
8	10,000	250		60	15.3	11.2	1.10
9				65	16.7	11.2	1.11
10				70	18.1	11.2	1.15
11				75	19.6	11.2	[a]
12	25,000	346		60	14.0	9.7	1.09
13				65	15.3	9.7	1.09
14				70	16.6	9.7	1.09
15				75	17.9	9.7	[a]

[a] Nominal cases.

Table 4 Vortex force on drogue

Altitude, ft	Velocity, knots	y_{end}, ft	V_0 at vortex eye, fps	Force, lb. $SC_D = 1.34$ at a	at y_{end}	$SC_D = 1.75$ at a	at y_{end}
S.L.	200	11.5	161	41.3	0.20	53.9	0.26
	265	7.7	124.8	24.8	0.27	32.4	0.35
	330	5.5	100	15.9	0.33	20.8	0.43
10,000	200	14.0	215	54.4	0.18	71.0	0.23
	250	9.9	176	36.5	0.24	47.6	0.31
	300	8.1	146	25.1	0.24	32.8	0.32
	330	7.0	133	17.8	0.27	23.3	0.36
25,000	205	18.0	351	88.2	0.17	115.2	0.23
	250	14.4	273	53.3	0.18	69.7	0.23
	292	11.9	246	43.3	0.20	56.6	0.26
	346	9.4	203	29.5	0.21	38.5	0.28

creasing weight reduces the gain but only slightly. Reducing hose length shifts the frequency peaks to a higher-frequency region but with reduced gain. In all cases the gain for 1.3 Hz and 2.1 Hz was less than unity.

Table 5 gives the solution of the perturbation equations for 4 nominal flight conditions. The solutions were obtained for sine inputs and for 1-cosine inputs as well. A closed-form solution by separation of variables was calculated for steady-state sine input cases. This was verified by a numerical solution for the same cases. The numerical method was used to solve the 1-cosine cases for a single pulse input. The length of the pulse was from 0 to 2π rad. In case 4, whipping occured as the hose is very long, 75 ft. The sensitivity of the design to wind gust disturbance is shown in Table 6.

A graphical presentation of the results for some of the cases (1, 2, 3, 4) of Table 5, is given in Figs. 4-7. Case 3 for a sine input is shown in Fig. 4. The amplitudes are small as the numerical solution is still in the transient dominant solution phase. Figure 5 shows the dynamic response for a 1-cosine input for case 3 as well. The slight whipping of the end is expected for this input.

The Table 6 sensitivity results show that reducing the wind disturbance magnitude to half approximately halves the sine input response. The most dramatic changes were obtained by varying the hose lengths. This was true for both hoses, and indicates a technique that might easily be applied to improve the design characteristics (if other requirements permit this change). Change of drogue weight had only a slight effect, which matches the static trail position sensitivity results. Similarly the system is not too sensitive to changes in C_DS of the drogue.

Figure 6 shows the dynamic response for case 1, for a sine input for a half-cycle of operation. (Case 1 is at sea level, 265 knots, hose 1). Figure 7 shows the dynamic response for case 1 for the 1-cosine pulse input.

There is an effect of tuning to the gust wavelength. For example, a change of 0.75 ft in gust wavelength results in a change of 1.18 ft in response (for case 3, see Table 5). It is fair to assume that the system has a characteristic "natural" wavelength for each set of design parameters and for each flight condition. This is an area for future study.

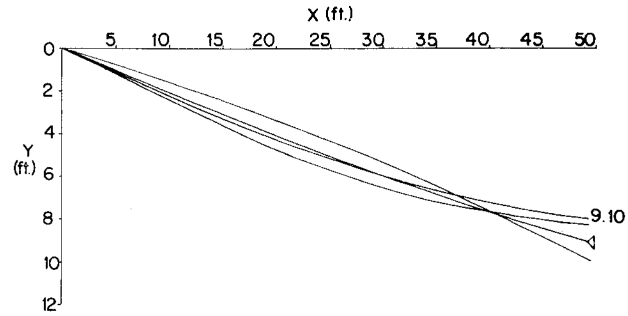

Fig. 4 Hose position at 25,000 ft, 346 knots for a sine input.

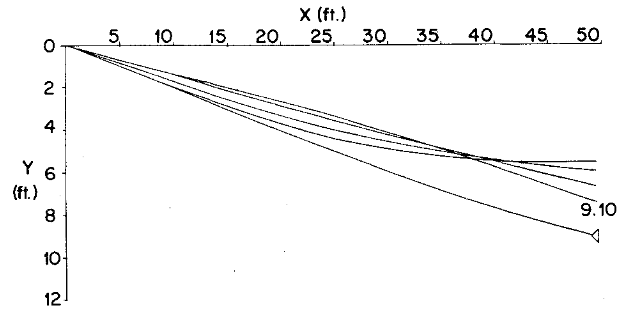

Fig. 5 Hose position at 25,000 ft, 346 knots for a 1-cosine input.

VI. Conclusions

The effect of the vortex was shown to be negligible. The wing vibration gain for hose 1 and at the drogue end showed a range from 6.8 at resonance at low frequencies to as low as 0.33 at the nonresonant high frequencies. At the critical frequencies of 1.3 and 2.1 Hz, the gain was 0.65 and 0.48, respectively. The wind gust effects show a range of maximum amplitude from 1.1 ft to 5.5 ft for the nominal sine wave. The superiority of hose 1 is evident in the pulse reactions, where it reacts about a third as much as hose 2.

Table 5 Wind gust disturbances

Case	Flight conditions: Altitude, ft	Flight conditions: Speed, knots	Conditions	Sine input	1-Cosine input	1-Cosine max. change in 1 second
1	S.L.	265	Hose 1	1.14	5.0	3.58
2	S.L.	265	Hose 2	5.54	15.7	9.41
3	25,000	346	Hose 1	3.48	3.6	2.93
4	25,000	346	Hose 2	2.23	11.1[a]	6.47

[a] Whips.

Table 6 Sensitivity of wind gust disturbance

Case	Flight conditions: Altitude, ft	Flight conditions: Speed, knots	Conditions	Sine input drogue max., ft	Sensitivity
1	25,000	346	Input wind amplitude reduced to ½ hose 1	1.74	+.95
2	25,000	346	Same as 1, hose 2	1.21	+.95
3	25,000	346	Drogue wt. increased to 57.8 lb, hose 1	3.24	...
4	25,000	346	Same as 3, hose 2	2.10	−.96
5	25,000	346	Same as 3, but 62.8 lb	2.31	−1.6
6	25,000	346	Same as 4, but 62.8 lb	1.77	−1.2
7	25,000	346	Hose length reduced to 40 ft., hose 1	0.50	4.24
8	25,000	346	Same as 7, but 45 ft	1.00	6.97
9	25,000	346	Hose length reduced to 55 ft, hose 2	0.80	2.45
10	25,000	346	Same as 9, but 65 ft.	1.07	4.14
11	25,000	346	C_DS of drogue reduced to 2 ft^2, hose 1	2.41	2.97
12	25,000	346	Same as 11, but hose 2	2.10	0.96

Table 7 Comparison of closed-form and numerical solutions – Case: wind gust, sine wave, steady-state

	Flight conditions						Steady-state amplitude	
	Altitude, ft	Speed, knots	Hose	ρ_H	s_0	D	Closed-form	Numerical
1	S.L.	265	1	1.858	50	2.0	1.16	1.14
2	S.L.	265	2	4.023	75	2.9375	5.62	5.54
3	25,000	346	1	1.858	50	2.0	4.13	3.48
4	25,000	346	2	4.023	75	2.9375	2.27	2.23
5	25,000	346	2	Increased W to 57.8			1.99	2.10
6	25,000	346	1	Decreased s_0 to 40.0			0.52	0.50
7	25,000	346	2	Decreased wind velocity to ½			1.13	1.21

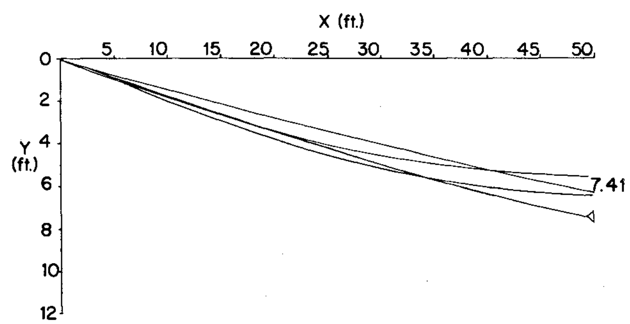

Fig. 6 Hose positions at sea level, 265 knots, for a sine input.

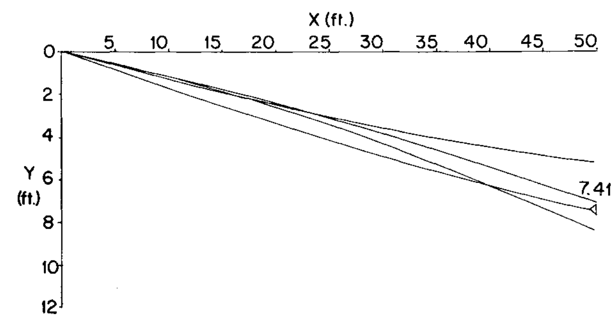

Fig. 7 Hose position at sea level, 265 knots for a 1-cosine input.

In each case sensitivity to design changes are calculated. The main points are as follows:

Static Trail Position

1) Hose 2 lowers y_{end} by about 2 ft below the hose 1 position.

2) The most sensitive parameter is hose length; shortening it yields higher y_{end}.

Wing Vibration

1) Large magnification of input amplitude occur at 1.0, 1.8, and 2.6 Hz.

2) Adding drogue weight has little effect.

3) Shortening the hose to 45 ft reduces the gain but shifts the peaks to the higher-frequency region, an unwanted effect.

Wind Gust Disturbance

1) Both at sea level and at 25,000 ft altitude, large motion of the drogue was encountered for the 1-cosine pulse input, but less so for hose 1 than hose 2.

2) High-altitude gusts (61.3 and 66 fps) were considered, and their probability of occurence may therefore be small, but as their relative effectiveness is proportional to the magnitude, the results regarding hose 1 and 2 are relatively correct.

3) Whipping was noticed in case 2 and 4, that is, hose 2.

4) Sensitivity calculations indicate once again that shortening the hose has the effect of reducing the drogue motion most effectively.

Combined Effects

Although the two disturbances, wing vibration and vertical wind gusts, are not independent, a rough estimate of their combined effect could be attained by root-sum-squaring their separate effects. This was not done because it was desired to obtain an estimate of each effect separately more than a single one-number answer.

Final Selection

Because of the superior response to vertical wind gust and the acceptable response to wing vibration, hose 1 was selected. The nominal configuration was chosen including a length of 50 ft. The sensitivity data was saved for further use, should it be necessary to improve the stability. Armed with the knowledge of the relative effects of hose length, drogue drag, and drogue weight, new design changes could be made intelligently. For example, one could improve the response by adding weight to the drogue since this does not impair wing vibration response but considerably reduces the wind-gust amplitude. Looking at the static sensitivity, we could immediately estimate the added droop at y_{end} due to this added weight. These changes to the system were not made yet. They will only be done if flight test results indicate the need for added stability.

Flight Test

Hose 1 at 50 ft length has been test flown and the first results are good. No deleterious vortex effect was noted, as the analysis indicated. It was a mild day and there were no strong wind gusts, but the system was steady in any event. Future fully instrumented test flights are planned that will allow validation of the analysis.

Future Tasks

A more complete performance optimization could now be conducted. This would use the programs developed in this report as subroutines to a merit function reflecting the design goal and a search routine such as gradient, simplex, or a grid search to solve for the optimum design parameters.

Acknowledgment

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